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PHOTOMETRIC ELEMENTS OF BOSS 5070

By GERALD E. KRON

ABSTRACT

The light-variation of Boss 5070 has been observed over a period of seven years, from 1923 to 1930, with the photoelectric photometer of the Washburn Observatory. Because of the length of the period, 12.4 days, and the shallowness of the minima, it was necessary to take a large number of readings over a long interval of time before enough data were at hand to determine a satisfactory light-curve.

The system is composed of a large B2 star with a smaller companion of greater surface intensity. The B2 star, because of its size, gives out so much more light than its companion that only one spectrum is visible. The minimum light-ratio and the large eccentricity of the orbit combine to remove to a large extent the indeterminateness of a solution based upon eclipses of only a tenth of a magnitude.

Spectroscopic elements of the eclipsing variable Boss 5070 were first published in 1920 by W. E. Harper.[1] Since then these elements have been revised, and the present photometric solution of the star was carried out with the help of new elements, received in April, 1935, from Dr. Harper. The spectrum is single and is of class B2.

The large value of the mass function of the star indicated that it might be an eclipsing variable, and it was therefore placed on the observing program of the Washburn Observatory. Observations were started in June, 1923, and on the first night the star was caught at secondary minimum. The measures were continued at times until 1930, with readings on sixty-four nights by Mr. Huffer and on seventeen nights by Mr. Stebbins. The data for the two comparison stars

[1] *Pub. A.A.S.*, 4, 219, 1920.

are given in Table I. Both were constant within the errors of the observations and were therefore used continually during the entire period of observation. A neutral shade was used on 22 Cygni.

TABLE I

HR	Star	Vis. Mag.	Spectrum	Remarks
7567	Boss 5070	5.62–5.74	B2	
7613	22 Cygni	4.87	B3	Shade 0.86
7628	....	5.43	B3	

The observational data are given in Table II. The phase was computed from the elements:

$$\text{Min.} = \text{J.D. } 2423587.145 + 12^{d}.4262 \cdot E \ .$$

Each observation consisted of two complete measures of each star, with four or more readings each. The Δ mag. in the fourth column is taken in the sense of mean brighter than Boss 5070. When the average deviation of the measures of a set is greater than ± 0.020 mag., that fact is indicated in the "remarks" column.

In the course of the work three photoelectric cells were used:

J.D. 2423592–4034........QK 203
4052–4117........QK 301
4285–6241........QK 302

In forming the normals in Table III a systematic correction of -0.023 mag. has been applied to the Δ mag. of the series with the first cell; the other two cells had practically the same characteristics. The residuals of the normals were determined graphically from the final light-curve in Figure 1. The probable error of one normal is ± 0.0060 mag.

A solution was first attempted by making use of Russell's method without any alterations. It was soon evident that there was a great amount of indeterminateness, and the eccentricity was so large (0.22) that its square could not be neglected. When the square of the eccentricity is taken into account in Russell's method for the solution

of eccentric orbits, the equations become very complicated, so a cut-and-try method of solution, using the spectroscopic elements, was adopted. The spectroscopic values of e and ω, along with the depths

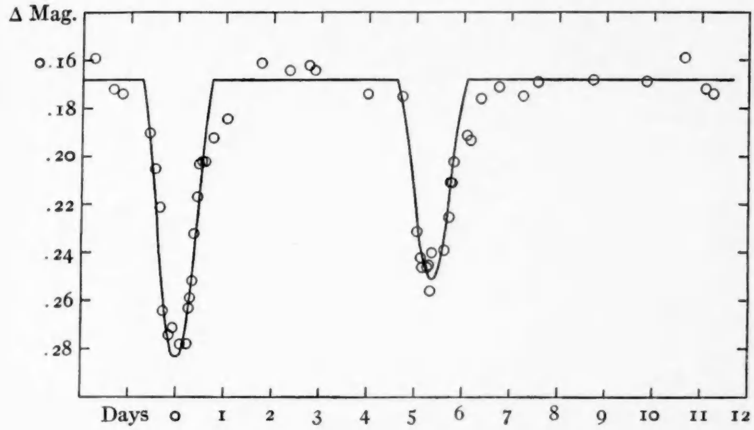

FIG. 1.—The light-curve of Boss 5070

of the minima and the times of beginning and ending of the primary eclipse, formed the most important quantities which had to be satisfied by the elements.

The solution was begun by solving the following equations[2] for k, α_1, and α_2:

$$\frac{1-\lambda_1}{\alpha_1}+\frac{1-\lambda_2}{k^2\alpha_2}=1\;, \tag{1}$$

$$\frac{(p_2-p_1)k}{2+(p_1+p_2)k}=e\sin\omega\;. \tag{2}$$

The value of $e \sin \omega$ is taken from the spectroscopic elements; the depths of the minima, or λ_1 and λ_2, are known from the photometric curve; the p's are Russell's p-functions. The two equations were solved by approximations. First a value of k was selected; then such values of the α's were found that both equations were satisfied. At first sight the amount of labor involved appears large, but the process may be aided by solving the first equation for one of the α's and by

[2] *Ap. J.*, **36**, 58, 1912.

TABLE II

Photoelectric Observations of Boss 5070

J.D. 242+	Phase	HR 7628 > 22 Cyg.	Mean > Boss 5070	Remarks
3592.820...	5^{d}672	0^{M}128	0^{M}266	± .025, 1923
3600.744...	1.173	.086	.181	
10.738...	11.167	.118	.213	
18.646...	6.649	.106	.179	
20.651...	8.614	.106	.202	
44.736...	7.886	.103	.182	
48.836...	11.986	.098	.207	
3725.616...	1.783	0.113	0.207	Smoke
3931.803...	9.151	0.122	0.187	1924
35.803...	0.724	.111	.225	
.815...	0.736	.112	.229	
52.711...	5.206	.104	.260	
.725...	5.220	.120	.266	
.756...	5.251	.140	.259	
.767...	5.262	.109	.281	
53.656...	6.151	.088	.196	
.669...	6.164	.096	.230	
.694...	6.189	.083	.224	1 set
77.660...	6.303	.080	.292	
.672...	5.315	.079	.280	± .025
83.703...	11.346	.122	.182	
.715...	11.358	.095	.204	
84.650...	12.293	.088	.293	
.662...	12.305	.090	.332	
.675...	12.318	.100	.320	
.692...	12.335	.075	.280	± .023, Smoke
.699...	12.342	.066	.300	Smoke
.825...	0.042	.095	.318	
.837...	0.054	.095	.305	
91.824...	7.041	.093	.204	
.836...	7.053	.111	.210	
4014.722...	5.086	.088	.266	
.737...	5.101	.082	.266	
.747...	5.111	.095	.272	
20.758...	11.122	.097	.186	
.769...	11.133	.093	.188	
22.588...	0.526	.105	.250	
.599...	0.537	.108	.234	± .024
26.723...	4.661	.110	.214	
.732...	4.670	.095	.188	
32.706...	10.644	.079	.202	Poor
34.685...	0.197	.066	.290	± .028
.695...	0.207	.096	.308	
.703...	0.215	.087	.306	
.712...	0.224	.084	.310	
.720...	0.232	0.121	0.288	
52.635...	5.721	0.115	0.236	± .035, New cell
.649...	5.735	.118	.209	
.666...	5.752	.131	.232	
57.697...	10.783	.170	.146	
59.619...	0.278	.114	.265	
.649...	0.308	.131	.256	
.669...	0.328	.142	.248	
.762...	0.421	.126	.220	
.786...	0.445	.120	.210	
61.618...	2.277	.183	.180	
.637...	2.296	.161	.169	
64.590...	5.249	.120	.245	
.610...	5.269	.123	.232	
.628...	5.287	.113	.241	
.650...	5.309	.115	.244	
.672...	5.331	.131	.236	
.690...	5.349	.133	.236	
.710...	5.369	0.132	0.237	

J.D. 242+	Phase	HR 7628 > 22 Cyg.	Mean > Boss 5070	Remarks
4064.758...	5^{d}417	0^{M}132	0^{M}234	
66.714...	7.373	.146	.150	
72.558...	0.791	.127	.196	
.577...	0.810	.135	.162	
75.584...	3.817	.142	.192	
.624...	3.857	.118	.158	
76.565...	4.798	.179	.169	
.701...	4.934	.178	.206	± .022
81.554...	9.787	.157	.157	
.568...	9.801	.122	.164	
82.606...	10.839	.128	(.127)	Discordant
.622...	10.855	.142	.164	
83.599...	11.832	.132	.177	
.614...	11.847	.149	.188	
84.551...	0.358	.133	.228	
.565...	0.372	.097	.224	
.578...	0.385	.149	.218	
.592...	0.399	.150	.225	
.604...	0.411	.133	.218	
.629...	0.436	.148	.216	
.646...	0.453	.163	.198	
.662...	0.469	.118	.200	
86.585...	2.392	.134	.164	
90.566...	6.373	.130	.172	
.580...	6.387	.160	.180	
91.558...	7.365	.148	.172	
.572...	7.379	.132	.182	
4108.585...	11.966	.129	.188	
.606...	11.987	.139	.198	
14.577...	5.532	.125	.242	1 set
.606...	5.566	.144	.262	
17.593...	8.548	.152	.166	
.612...	8.567	0.146	0.164	
4285.741...	2.729	0.153	0.164	1925
.750...	2.738	.156	.162	
95.724...	0.286	.151	.258	
.734...	0.296	.131	.253	
.755...	0.317	.129	.244	
.764...	0.326	.150	.254	
4307.684...	12.246	.138	.279	
.697...	12.259	.144	.260	
.709...	12.271	.146	.262	
.740...	12.302	.154	.268	
.751...	12.313	.136	.271	
76.760...	6.764	.136	.178	
.772...	6.776	.146	.180	
84.794...	2.372	.137	.154	
.807...	2.385	.121	.151	
94.799...	12.377	.133	.264	
.811...	12.389	.143	.256	
97.714...	2.866	.128	.166	
.726...	2.878	.130	.158	
4417.641...	10.367	.132	.148	
22.600...	2.900	.146	.162	
.611...	2.911	.150	.171	
75.546...	6.141	.139	.182	
.557...	6.152	.136	.199	
.571...	6.166	0.154	0.188	
4686.764...	6.113	0.154	0.189	1926
.775...	6.124	.151	.190	
4691.702...	11.051	.162	.192	
.716...	11.165	.146	.180	
93.721...	0.644	.154	.197	
.732...	0.655	.150	.199	
4748.712...	5.930	.133	.202	
52.698...	9.916	.146	.191	
.709...	9.927	0.158	0.178	

TABLE II—*Continued*

J.D. 242+	Phase	HR 7628 > 22 Cyg.	Mean > Boss 5070	Remarks
4752.719...	9^d937	0^M161	0^M164	
54.587...	11.805	.143	.206	
.706...	11.924	.155	.199	
.717...	11.935	.155	.206	
.756...	11.974	.155	.216	
.768...	11.986	.156	.216	
55.588...	0.380	.192	.242	
.699...	0.491	.168	.207	
.710...	0.502	.141	.194	
.721...	0.513	.140	.206	
.753...	0.545	.156	.204	
.765...	0.557	.174	.200	
68.655...	1.021	.170	.183	
.665...	1.031	.151	.178	
.674...	1.040	.171	.190	
69.654...	2.020	.157	.165	
.665...	2.031	.169	.160	
72.649...	5.015	.173	.225	
.681...	5.047	.155	.240	
79.637...	12.003	.182	.220	
.649...	12.015	.149	.239	
97.634...	5.148	.149	.260	
.644...	5.158	0.177	0.259	
5071.638...	5.775	0.154	0.222	1927
.649...	5.786	.147	.206	
.660...	5.797	.140	.214	
.672...	5.809	.150	.208	
77.699...	11.836	.140	.190	
.710...	11.847	.133	.187	
5140.641...	0.221	.153	.267	
.651...	0.231	.123	.268	
.660...	0.240	.125	.268	
.671...	0.251	.161	.263	
.699...	0.279	.141	.252	
.708...	0.288	.157	.262	
.718...	0.298	.151	.252	
45.672...	5.252	.137	.246	
.683...	5.264	.141	.247	
.690...	5.270	......	(.257)	1 set
.697...	5.277	.151	.258	
70.556...	5.284	.133	.247	
.564...	5.292	.144	.261	
.573...	5.301	.135	.256	
.583...	5.311	.170	.252	
72.631...	7.359	.134	.165	
.639...	7.367	.138	.170	
.652...	7.380	0.152	0.173	
5438.685...	0^d037	0^M154	0^M269	1928
56.789...	5.714	.133	.216	
.797...	5.722	.143	.214	
.808...	5.733	.141	.218	
.837...	5.762	.135	.206	
.847...	5.772	.149	.206	
.856...	5.781	.154	.189	
.866...	5.791	.156	.204	
.876...	5.801	.158	.198	
.887...	5.812	.162	.184	
63.643...	0.142	.155	.256	
.654...	0.153	.162	.286	
.744...	0.243	.136	.253	
.758...	0.257	.135	.260	
67.624...	4.123	.138	.171	
.637...	4.136	.151	.175	
68.610...	5.109	.160	.236	
.619...	5.118	.144	.247	
.629...	5.128	.150	.249	
94.692...	6.338	.144	.180	
.702...	6.348	.148	.172	
5513.638...	0.432	.146	.206	
.655...	0.449	.160	.198	
.723...	0.517	.137	.194	
.733...	0.527	.136	.191	
55.587...	5.102	.161	.232	
.596...	5.111	.154	.247	
.606...	5.121	0.141	0.226	
6120.757...	11.093	0.143	0.163	1930
.767...	11.103	.176	.164	
21.758...	12.094	.133	.262	
.767...	12.103	.158	.266	
.777...	12.113	.148	.247	
.787...	12.123	.156	.272	
.799...	12.135	.154	.258	
27.694...	5.604	.140	.230	
.707...	5.617	.154	.234	
.759...	5.669	.168	.225	
.772...	5.682	.144	.225	
.814...	5.724	.156	.210	
.828...	5.738	.149	.210	
.842...	5.752	.150	.210	
48.678...	1.736	.147	.155	
.692...	1.750	.153	.154	
52.633...	5.691	.137	.236	
6236.698...	2.772	.171	.167	
.708...	2.782	.143	.154	
41.622...	7.696	.142	.180	
.631...	7.705	0.167	0.168	

making an accurate plot of the p-function. A minimum value of k is set by equation (1) when α_2 is equal to unity. A maximum value of k may be worked out from the fact that the ratio of the lights of the components must be greater than 3 to 1.

These two values of k were, respectively, 0.34 and about 0.40. A series of values of k between these two limits and the corresponding values of the α's were computed. It now remained to find the radius of the larger component, r_1, and the inclination of the orbit for each of these values of k. This was done by making use of two relation-

ships between these two quantities, one derived from the middle of the primary eclipse and the other from the end. These equations are

$$(\text{Middle of eclipse})\ \delta_1 = r_1(1+p_1k) = r\cos i\ , \tag{3}$$

$$(\text{End of eclipse})\ \delta_2 = r_1^2(1+k)^2 = r^2 + r^2\sin^2 i[\cos^2(v+\omega)-1]\ , \tag{4}$$

where r is the radius vector or distance between centers. Equations (3) and (4) may be solved simultaneously for r_1 and i. The light-

TABLE III

NORMAL MAGNITUDES

Phase	Δ Mag.	Resid.	No. Obs.	Phase	Δ Mag.	Resid.	No. Obs.
0^d086	0^M278	0^M003	5	5^d356	0^M240	-0^M010	5
0.213	.278	+ .009	5	5.618	.239	+ .004	6
0.239	.263	.000	5	5.706	.225	.000	5
0.278	.259	+ .002	5	5.736	.211	− .008	5
0.309	.252	+ .008	5	5.768	.211	.000	5
0.365	.232	.000	5	5.799	.202	− .002	6
0.420	.217	− .002	5	6.077	.191	+ .020	4
0.461	.203	− .016	5	6.162	.193	+ .026	5
0.517	.202	− .010	5	6.362	.176	+ .008	4
0.588	.202	+ .008	5	6.730	.171	+ .003	3
0.765	.192	+ .024	4	7.237	.175	+ .007	5
1.031	.184	+ .017	3	7.570	.169	+ .001	6
1.746	.161	− .007	6	8.720	.168	.000	4
2.344	.164	− .004	5	9.874	.171	+ .003	5
2.755	.162	− .006	4	10.662	.159	− .009	4
2.889	.164	− .004	4	11.087	.172	+ .004	5
3.983	.174	+ .007	4	11.251	.174	+ .007	4
4.710	.175	.000	3	11.833	.190	+ .001	5
5.037	.231	+ .003	5	11.957	.205	− .002	5
5.110	.242	+ .008	5	12.071	.221	− .018	5
5.152	.246	+ .008	5	12.144	.264	+ .008	5
5.247	.246	+ .001	5	12.286	.274	− .002	5
5.276	.245	− .001	5	12.347	0.271	−0.010	6
5.303	.256	+ .008	5				

curve was then computed from these elements, and the solution which gave the most satisfactory shape of curve and width of secondary minimum was selected. The value $k=0.36$ seemed to be the best, and this was adopted. The larger component is behind at primary, and has the lesser surface intensity, as in Figure 2. It is this combination of a relatively large star and a smaller and more intense one which makes the secondary minimum almost as deep as the primary, even though the eclipsed area during secondary is less than half of that during primary. The larger of the two stars gives out so much more light than the smaller that the spectrum of the

larger component alone is recorded. A single spectrum indicates that the ratio of the light of the components must be at least 3 to 1—a fact which contributes toward a knowledge of the limits within which the value of k lies. The eccentricity of the orbit is large and is known with considerable accuracy from the spectroscopic solution. The limiting ratio of the light and the large eccentricity together remove most of the indeterminateness from an otherwise hopeless problem.

It is obvious from an inspection of the light-curve that there are still possible effects that have not been taken into consideration in this solution, namely, a periastron effect of perhaps 0.01 mag., the ellipticity of the figure, and the darkening at the limb. It was considered best not to overwork the data at hand by trying to reduce residuals which are already quite small.

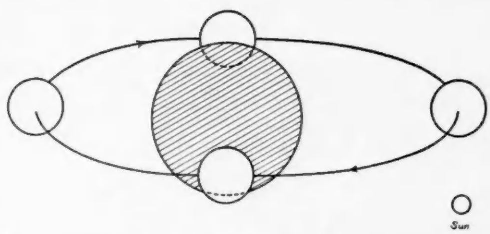

FIG. 2.—The system of Boss 5070

In order to solve completely for the masses, parallax, densities, etc., it was necessary to assume a value for either the mass or the absolute magnitude of one of the components. Accordingly, the value of -2.0 mag. was assumed for the absolute visual magnitude of the B2 component. A correction of -1.49 mag. was applied to this to give an absolute bolometric magnitude of -3.49 mag. and a mass of 9.7 from Eddington's mass-luminosity relationship. The mass of the other component, 7.3, was computed from this and the mass function. The value a_1 was known; a_2 was computed from a_1 and the mass ratio, thus leading to the size of the semi-major axis of the orbit and the diameters and densities of the components. The absolute visual magnitude was changed to absolute photoelectric magnitude by applying the correction -0.04 mag., from which the absolute photoelectric magnitude of the smaller component was found by making use of the previously determined photoelectric light-ratio. The parallax of 0."0029 follows from the assumed absolute magnitude.

The elements are summarized in Table IV. The period, 12.4262 days, derived by Mr. Huffer from our data for the interval of seven

years, differs slightly from the new one by Harper, 12.4255 days, but there is only a small inconsistency in using our period with his spectroscopic elements.

TABLE IV

PHOTOMETRIC ELEMENTS OF BOSS 5070		
Period	P	12^d4262
Primary minimum, J.D.	t_1	2423587.145
Phase of secondary minimum	t_2-t_1	5^d350
Amount of eclipse, primary	α_2	0.950
Amount of eclipse, secondary	α_1	0.389
Ratio of radii	k	0.36
Loss of light at t_1	$1-\lambda_2$	0.100
Loss of light at t_2	$1-\lambda_1$	0.073
Light of larger body	L_1	0.812
Light of smaller body	L_2	0.188
Ratio of surface brightnesses (photoelectric)	J_1/J_2	0.56
Inclination of orbit	i	$72^\circ16$
Radius of larger body	r_1	0.347
Radius of smaller body	r_2	0.125
SPECTROSCOPIC ELEMENTS BY HARPER		
Eccentricity of orbit	e	0.222
Half-range of velocity	K	94.01
Longitude of periastron	w	$115^\circ75$
Mass function	f	0.994
COMBINED ELEMENTS		
Mass of larger body	m_1	9.7*
Mass of smaller body	m_2	7.3
Radius of larger body	r_1	9.6
Radius of smaller body	r_2	3.4
Density of larger body	ρ_1	0.011
Density of smaller body	ρ_2	0.178
Distance between centers	a	38.2×10^6 km
Absolute photoelectric magnitude	M_1, M_2	$-2.04, -0.45$
Parallax	π	$0''0029$
Light of larger body	L_1	870
Light of smaller body	L_2	200

* Sun = 1.

I am greatly indebted to Mr. Stebbins and to Mr. Huffer for permission to use the observational material and for general advice during the course of this work.

WASHBURN OBSERVATORY
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ABSORPTION LINES DUE TO AN EXPANDING STAR*

By O. C. Wilson

ABSTRACT

The modification of the shape of an absorption line produced by an expansion or contraction of a star is investigated. It is shown that Carroll's approximation is valid only for small velocities and that for large velocities important changes in the shape and strength of a line occur. Applications to giant stars and Cepheid variables are discussed briefly and to Wolf-Rayet stars and novae with more detail. A method is indicated by which, if the geometrical explanation of certain features in the spectrum of Nova Herculis is considered valid, the linear dimensions of the star may be computed. A preliminary calculation gives for the radius of the photosphere of Nova Herculis in its earlier stages a value of the order of 100×10^6 km.

The subject of this paper is by no means new. The modification of the shape of an absorption line due to an expansion or contraction of a star was investigated as long ago as 1919 by Shapley and Nicholson,[1] and more recently by Carroll.[2] There are two reasons, however, why a reinvestigation of this matter does not appear to be out of place at the present time. First, the results of Shapley and Nicholson, while entirely correct, are presented in the form of curves showing the modified and unmodified lines, but these curves are not provided with vertical scales. Hence they tend to emphasize only the asymmetry of the modified line without calling attention to the changes in the absolute intensity within it. As will be seen, the latter may be quite large, even for rather modest velocities. Second, there is at first glance an apparent contradiction between the results of the two papers. Thus, Shapley and Nicholson derive an appreciable change in shape for rather small velocities, of the order of those observed in Cepheid variables, while, according to Carroll, the only effect should be to shift the line bodily, leaving its shape and intensity unaffected. This result of Carroll's has, for instance, led Gerasimovič to make the statement[3] that in Nova Aquilae 1918 no change in the shapes of the absorption lines would be expected unless

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 526.

[1] *Mt. W. Comm.*, No. 63; *Proc. Nat. Acad.*, **5**, 417, 1919.

[2] *M.N.*, **88**, 548, 1929.

[3] *Zs. f. Ap.*, **7**, 335, 1933.

they were, intrinsically, of infinitesimal width. In view of these facts it seems worth while to re-examine the whole question.

Let I be the emission per unit area of the stellar surface in the direction of the normal to the surface for wave-lengths in the continuous spectrum closely adjacent to the line in question. Then within the line, at distance $\Delta\lambda$ from its center, the emission may be written

$$i(\Delta\lambda)=If(\Delta\lambda)\,. \tag{1}$$

If we view the element of surface from a direction which makes an angle ϑ with the normal, we have, more generally,

$$i(\Delta\lambda,\,\vartheta)=I\psi(\vartheta)f(\Delta\lambda,\,\vartheta)\cos\vartheta\,. \tag{2}$$

The function $\psi(\vartheta)$ takes account of limb darkening in the continuous spectrum, while the appearance of ϑ in f allows for the possibility of a change in the form or intensity of the line over the disk. In the sun, for example, $\psi(\vartheta)$ is very nearly equal to $\frac{2}{5}+\frac{3}{5}\cos\vartheta$, while the dependence of f on ϑ seems to vary with the line under consideration and is never very great,[4] except, of course, just at the limb itself.

From equation (1), the light-ratio in the line at $\Delta\lambda$, for a small element at the center of the disk, is

$$r(\Delta\lambda)=f(\Delta\lambda)\,, \tag{3}$$

and from (2), for integrated light,

$$r(\Delta\lambda)=\frac{\int\psi(\vartheta)f(\Delta\lambda,\,\vartheta)\cos\vartheta d\sigma}{\int\psi(\vartheta)\cos\vartheta d\sigma}\,, \tag{4}$$

where $d\sigma$ is the element of surface. From (4) it is perhaps worth noting that for lines in which f is independent of ϑ, the same line contour is obtained from the integrated light as from the center of the disk, regardless of limb darkening in the continuous spectrum.

Now imagine that the star begins to expand radially with velocity v, everything else remaining as before. With ϑ as defined above, the radial velocity for any element is $u_1=-v\cos\vartheta$. Consider the

[4] R. v. d. R. Woolley, *M.N.*, **93**, 708, 1933.

point in the spectrum at distance u, measured in velocity units, from the normal position of the line. At this point the intensity due to the element of surface having radial velocity u_1 is

$$di(u)=I\psi(\vartheta)f\left[\frac{\lambda}{c}(u-u_1),\ \vartheta\right]\cos\vartheta d\sigma\,, \tag{5}$$

and the total emission at u is, accordingly,

$$i(u)=I\int f\left[\frac{\lambda}{c}(u-u_1),\ \vartheta\right]\psi(\vartheta)\cos\vartheta d\sigma\,. \tag{6}$$

To get $r(u)$ we divide by the intensity of the adjacent continuous spectrum. It is easily seen that the latter, to terms of the order v/c, is the same as before, namely,

$$I\int\psi(\vartheta)\cos\vartheta d\sigma\,.$$

Hence

$$r(u)=\frac{\int f\left[\frac{\lambda}{c}(u-u_1),\ \vartheta\right]\psi(\vartheta)\cos\vartheta d\sigma}{\int\psi(\vartheta)\cos\vartheta d\sigma}\,. \tag{7}$$

For the sake of simplicity, assume that f is independent of ϑ and that $\psi(\vartheta)=1$. Then, since

$$\left.\begin{aligned} d\sigma&=2\pi R^2\sin\vartheta d\vartheta\,,\\ u_1&=-v\cos\vartheta\,,\\ du_1&=v\sin\vartheta d\vartheta\,,\end{aligned}\right\} \tag{8}$$

where R is the radius of the star, we find from (7)

$$r(u)=\frac{2}{v^2}\int_0^{-v}f\left[\frac{\lambda}{c}(u-u_1)\right]u_1du_1\,. \tag{9}$$

If we expand in a Taylor series and retain only the first two terms,

$$f\left[\frac{\lambda}{c}(u-u_1)\right]=f\left[\frac{\lambda}{c}u\right]-\frac{\lambda}{c}u_1f'\left[\frac{\lambda}{c}u\right]\,.$$

Hence, by (9),

$$r(u)=\frac{2}{v^2}\left\{f\left[\frac{\lambda}{c}\,u\right]\frac{v^2}{2}+f'\left[\frac{\lambda}{c}\,u\right]\frac{v^3}{3}\right\}\,.$$

Considering this expression as the first two terms of Taylor's expansion, we have

$$r(u)=f\left[\frac{\lambda}{c}\left(u+\tfrac{2}{3}v\right)\right]\,.$$

This result is, essentially, Carroll's approximation and shows at once that the line is merely shifted to the violet by an amount $\frac{2}{3}\,v$,

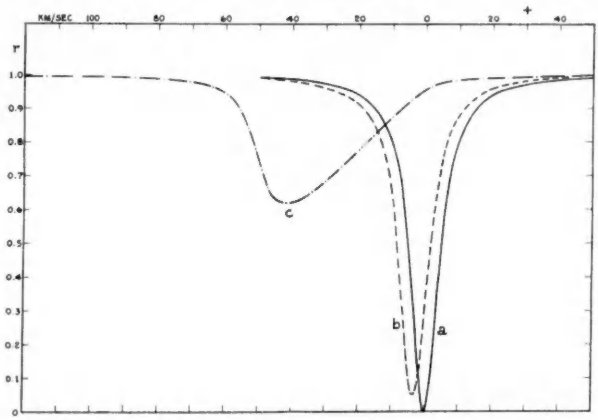

FIG. 1

but otherwise is unchanged. It is unsafe, however, to draw conclusions from such a result without first examining the quality of the approximation.

The simplest method of investigating the validity of Carroll's approximation is to choose unmodified lines like those actually observed in the sun and in normal stars and, by means of the rigorous equation (9), compute the modification produced for a series of velocities.

As a first example let us adopt as the expression for the unperturbed line contour

$$r(\Delta\lambda)=\frac{1}{1+(a\Delta\lambda)^{-2}}\,, \tag{10}$$

where a is an arbitrary constant. This is the type of line which would be produced by a simple scattering atmosphere, transparent everywhere except near the line frequency, and possessing an Unsöld scattering coefficient in that region. It is probably a good approximation to the true shapes of many stellar and solar absorption lines, except, perhaps, at the very center, and should suffice for our purpose. For

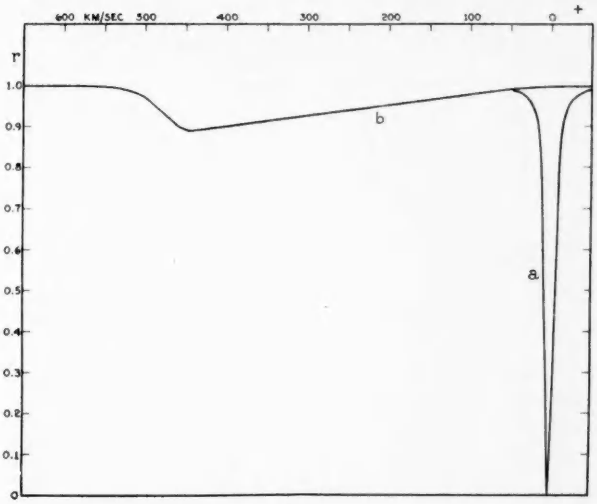

FIG. 2

$a=10$, the line will have a total absorption of about one-third of an angstrom and thus be normal in every respect.

Setting $10\lambda/c=\gamma$, we find from (10) and (9)

$$r(u)=1-\frac{1}{v^2\gamma^2}\left[\ln\left\{\frac{\gamma^2(u+v)^2+1}{\gamma^2u^2+1}\right\}-2u\gamma\tan^{-1}\frac{\gamma v}{1+\gamma^2u(u+v)}\right] \quad (11)$$

for the contour of the line when the velocity of expansion is v. For simplicity in calculation we take λ to be 6000 A, giving $\gamma=0.2$.

From equation (11) we compute the line shapes for values of v equal to 5, 50, and 500 km/sec. These velocities of expansion are of the order of magnitude of those to be expected: (1) for certain lines in giants and supergiants,[5] (2) for Cepheid variables, and (3) for novae. The results are plotted in Figures 1 and 2. In Figure 1 curves

[5] Adams and MacCormack, *Mt. W. Contr.*, No. 505; *Ap. J.*, **81**, 119, 1935.

a, b, and c are, respectively, the normal line, the result for $v=5$ km/sec., and that for $v=50$ km/sec.; while in Figure 2, a is the normal line and b the line as modified by a velocity of expansion of 500 km/sec.

We see that for $v=5$ km/sec. Carroll's approximation is fairly good. The line is shifted by about the right amount ($\frac{2}{3}v$) and its shape and intensity are not greatly altered. But for the other two velocities the approximate formula fails and can lead only to erroneous conclusions.

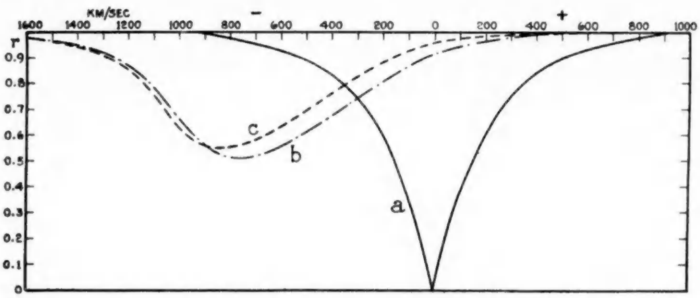

FIG. 3

Figure 3 shows the results when the unperturbed line is wide and of the form

$$r(\Delta\lambda)=1-e^{-\beta|\Delta\lambda|} ,$$

similar to that of the strong hydrogen lines in early-type stars. Curve a is the original line; b, the result for a velocity of expansion of 1000 km/sec.; and c, a recalculation of b for the case in which the darkening at the limb is the same as in the sun (coefficient of darkening $=\frac{3}{5}$).

APPLICATIONS

1. *Giant stars.*—The work of Adams and Miss MacCormack has shown that in certain giant stars there are systematic displacements of various lines which they have interpreted as being due to vertical currents in the stellar atmospheres, having velocities of the order of several km/sec. Theoretically, at least, it should be possible to check this interpretation by accurate measurement of the shapes of the

displaced lines. Actually, however, as a reference to Figure 1*b* shows, the modification for such velocities is probably too small to be measured, even on coudé spectrograms.

2. *Cepheid variables.*—In the case of Cepheids the situation is more hopeful. Without going into questions of resolving power at present, the fact that the 15-foot camera of the coudé spectrograph yields spectra in which moderately strong lines in the iron-arc comparison spectrum can be clearly separated when their wave-lengths differ by only a tenth of an angstrom leads one to expect that an effect of the order of that shown in Figure 1*c* should be readily observed. It is hoped that coudé plates of Cepheids can be examined in the near future from the standpoint of line shape.

3. *Wolf-Rayet stars and novae.*—According to generally accepted ideas, the bright bands in the spectra of these objects arise in shells or envelopes of ejected matter. Whether or not this ejection is uniform over the stellar surface is not known. It is, however, simplest to assume that this is the case—an assumption supported to some extent by the fact that direct photographs of Nova Aquilae 1918 now show the star to be surrounded by an almost exactly circular disk of nebulosity.

Since the velocities involved must be large in order to account for the great widths of the emission bands, the spectra of Wolf-Rayet stars and novae should therefore be the most likely places in which to search for lines of the kind illustrated in Figures 2 and 3.

From published photographs and descriptions of Wolf-Rayet spectra[6,7] it appears that there is a good, though not perfect, correlation between emission-band width and lack of absorption lines other than those of a broad shallow character. In general, spectra in which the emission bands are very wide have, in addition, a number of vague, diffuse features which can be variously interpreted as faint wide emission bands, wide shallow absorption lines, or a mixture of both. Beals has remarked that if the ejected atoms possess a wide range of velocities, the resulting absorption lines will be broad and shallow. Figure 2*b* shows, however, that if the atoms are ejected with a common large velocity uniformly over the star, they will also

[6] J. S. Plaskett, *Pub. D.A.O.*, **2**, 287, 1924.

[7] C. S. Beals, *ibid.*, **6**, 9, 1934.

produce a wide shallow absorption line. In fact, unless the unperturbed line is of considerable strength, the modified line will be so wide and shallow as to be unobservable. For instance, for the line used in Figures 2 and 3 and $v = 1000$ km/sec., $r(u)$ is unity to terms of order 10^{-3} for all values of u.

Therefore, the ejection hypothesis, by itself, gives a simple explanation of the lack, or scarcity, of observable absorption lines in these stars. The importance of differentiating observationally between Beals's interpretation and the present one is obvious. If, as in Beals's original argument, the lack of flat tops to the emission-band contours and the scarcity of sharp absorption features in the spectra of Wolf-Rayet stars are to be ascribed to a large velocity-range in the ejected atoms, then any absorption lines which are present

TABLE I

R	$\delta u/v$	R	$\delta u/v$
1.0	1.00	1.8	0.17
1.2	0.44	2.0	.13
1.4	0.29	3.0	.06
1.6	0.22	4.0	0.03

should be broadened more or less symmetrically about the position corresponding to the mean velocity of ejection. If, however, the absorption-line width is due to the fact that the atoms near the star's effective photosphere are all moving rapidly outward with a more or less common velocity, the line should be distinctly asymmetrical. The observed shapes of the emission bands could then be accounted for by velocity gradients in the envelope as suggested by Gerasimovič.[3]

We turn now to the question of novae. Why is it that in these objects we do not commonly find lines of the sort illustrated in Figure 2*b*? The purely geometrical argument would be as follows: Imagine that the ejected material is in the form of a spherical shell whose thickness is moderately small compared to its radius R. Let v be the velocity of ejection and let δu be the range in radial velocity over which absorption of the photospheric light may occur. Table I then

shows $\delta u/v$ as a function of R, the latter being expressed in terms of the radius of the photosphere as unity.

Hence if the velocity is large, say 1000 km/sec. or more, and the star has dimensions similar to those of an average star, a few hours should suffice to diminish $\delta u/v$ to the point where the absorption lines appear relatively narrow. As far as I am aware, the spectrum of a nova has never been obtained so early in its rise to maximum as to record the early stage of the ejection from the surface.

TABLE II

Plate	Date G.M.T.	Metallic Lines	K; λ 3933	*CN*; λ 3883
V 703....	Dec. 21.54	I fairly sharp; II absent	I sharp	Absent
706....	22.56	I fairly sharp; II absent	I sharp; II appearing on violet edge?	Very faint?
708....	23.55	I sharp; II very faint on violet edge of I?	I sharp; II on violet edge	Faint
712....	24.09	All diffuse and shallow	Wide; I and II blended?	Stronger
713....	24.55	All diffuse and shallow; some signs of duplicity?	Wide; I and II blended?	Strong
716....	25.09	All diffuse; some double?	Wide; I and II blended?	Strong
717....	25.55	II fairly sharp; I sharp and weak; some blends of I and II?	Wide; I and II blended?	Strong
722....	26.09	II sharper; I weak	Wide; I and II blended?	Strong
723....	26.55	II sharper; I nearly gone	Somewhat narrower?	Strong
725....	28.56	II sharp; I gone?	About same; violet wing extended?	Strong
726....	31.07	II sharp	II sharp; III on violet edge	Weaker
727....	Jan. 3.08	II sharp	II sharp; III wide but separated from II	Very weak
728....	6.07	II sharp	II and III separated; II sharp	Absent
733....	12.06	II sharp	III wide, well separated; III stronger and wider	
737....	16.07	II sharp	II and III about equally sharp; III stronger	Absent

With the average nova, however, successive outbursts seem to occur; and if the first is missed, one might expect to see broad shallow absorption lines appear whenever a new shell is ejected. But, as previously noted, unless these lines were intrinsically strong, a large velocity of ejection would so distort and weaken them that, owing to overlapping and blending with emission due to the previous ejections, they would escape detection.

Thus our best opportunity for observing lines of the sort discussed here would be in a nova of the less violent type, such as Nova Herculis 1934. Table II gives a brief description of certain features, including the character of the absorption lines, as they appeared to the writer in the spectrum of Nova Herculis between December 21, 1934, and January 16, 1935. The region examined extended from about λ 4100 to λ 3800. During this period, at least three separate

and distinct absorption components were present, denoted, respectively, by I, II, and III. The corresponding velocities, in round numbers, determined when the respective sets of lines were sharp, are -170, -315, and -700 km/sec. Component III appeared only in Ca^+ and H.

Let us attempt a geometrical interpretation of these facts. On December 21 the spectrum appeared much as it had for several days previously, and the lines were fairly sharp, owing to the smallness of $\delta u/v$. Sometime on December 22 the ejection of the shell which was to give rise to component II began. At first the lines due to this material would be shallow and wide and we should expect to see the first effects somewhat to the violet of the lines of component I. At about the same time λ 3883, CN, made its first appearance. That CN belongs with II is definitely proved by the similarity of their measured radial velocities.

The ejection of II continued during December 23 and 24, and by December 25 the lines of II could be clearly separated from those of I. During the next several days, I became progressively weaker, while the lines of II steadily increased in sharpness. Since II did not appear to change appreciably in quality after the plate of December 28, we may assume that maximum sharpness was attained on or about December 30. During this stage the CN bands, after maintaining a great strength more or less constantly for several days, quickly weakened and disappeared.

Considering the parallel set of changes in Ca^+, λ 3933, we find much the same situation. The main difference seems to be that because the I and II components in this case were inherently so much stronger and wider, they could not be clearly separated, while the weaker metallic lines were resolved.

About December 28 the K line began to show a wide, winglike extension to the violet, which we attribute to the beginning of the ejection of another shell, restricted, in this case, however, to Ca^+ and H. By January 3 components II and III of Ca^+ (and also of H) could be clearly separated, but III was much wider. Component III then gradually became sharper until, by January 16, it was similar to II.

Thus the very simple geometrical picture of successive ejections of thin spherical shells provides a good explanation of the sequences of

changes summarized in Table II. Of course the shells cannot really be thin. The ejection probably continues during some tens of hours, and, since it occurs at high velocity, the actual linear thickness of the shell must be considerable. Nevertheless, the general features of our interpretation should still hold.

These statements are not to be interpreted as implying that the elementary hypothesis thus far used is capable of giving a complete explanation of all the absorption phenomena observed in the spectrum of Nova Herculis. That such is not the case is indicated by only a cursory examination of the available material. This fact is, however, not surprising. There must be many factors, physical as well as geometrical, entering into the production of the spectrum, and it would be strange, indeed, to find only one of them, which, by itself, would suffice to explain all the observations. We wish merely to suggest that, for the epoch and the lines we have been considering, the geometrical hypothesis discussed in this paper may be the predominating one in producing the observed sequence of changes.

If we adopt this explanation of the observations, we are in a position to make an interesting and important calculation. We can compute the linear dimensions of the nova, and, provided we know the distribution of intensity in its continuous spectrum, its parallax. In what follows only round numbers and orders of magnitude will be used, the intention being to illustrate the method rather than to arrive at definitive values. More exact values can be obtained when more and better observations become available, if the method itself is found to be reliable.

Let R_0 be the radius of the photosphere and R the radius (some kind of a mean) of the shell. We shall assume, for the spectrograph used here, that as δu decreases, we are unable to see any further sharpening of a line after δu attains the value 20 km/sec. Consider component II of the metallic lines. The ejection began on, say, December 23.0. After December 28.6 the lines do not appear to sharpen appreciably. Hence we take $\delta u = 20$ km/sec. on December 29.0. For this component $v = 315$ km/sec. (assumed constant). Therefore, on December 29, $\delta u/v = 20/315 = 0.06$, and from Table I, $R/R_0 = 3.0$.

But, if the ejection began on December 23.0 and the velocity was constant, we have for December 29.0

$$R-R_0=6\times 86,400\times 315=163\times 10^6 \text{ km} .$$

From these two relations

$$R_0=82\times 10^6 \text{ km}=117 \odot .$$

Again, component III of Ca^+ first appeared on, say, December 29.0. Assume that it attained its maximum sharpness on January 14.0. Hence, for this date $\delta u/v=20/700=0.03$ and $R/R_0=4.0$. Likewise

$$R-R_0=17\times 86,400\times 700=1030\times 10^6 \text{ km} ,$$

and therefore

$$R_0=343\times 10^6 \text{ km}=490\odot .$$

The agreement between these two crude, preliminary computations is all that could be expected. Probably somewhat more weight should be given the estimate from the weaker metallic lines; hence 100×10^6 km for the radius of the photosphere is perhaps a fairly reliable order-of-magnitude measure of the dimensions of the nova at the epoch of the observations.

Some years ago the late S. R. Pike pointed out[8] that, in all probability, we must admit a very considerable increase in the size of a nova in order to account for the enormous observed increase in the emitted radiation. Incidentally, his calculated radius for Nova Aquilae 1918 at maximum is of just the same order as that which we have derived observationally for Nova Herculis. In the latter case, the fact that *CN* molecules are present in the ejected matter seems to argue against a tremendously high temperature, and it may well be that most, if not all, of the twelve or thirteen magnitudes of light-increase must be attributed to an increase in the radiating surface. If so, a radius of 100×10^6 km near maximum is quite reasonable.

In conclusion, it may be worth noting, for completeness, that if the distribution of radiation in the continuous spectrum of Nova

[8] *M.N.*, **89,** 538, 1929.

Herculis were the same as in the sun, a radius of 100×10^6 km would yield an absolute magnitude of -5.9. If we take the apparent magnitude during the latter part of December as $+2.0$ in round numbers, we find for the parallax $0\rlap{.}''0026$. When observed values of the color temperature become available, the absolute magnitude and parallax can of course be computed more accurately.[9]

Carnegie Institution of Washington
Mount Wilson Observatory
May 1935

[9] *Note added in proof:* Since the foregoing was written, two papers have appeared which have a direct bearing on the results for Nova Herculis. In the first, Petrie (*Ap. J.*, **81**, 482, 1935) finds that during the period December 30, 1934, to February 27, 1935, the color temperature of the nova remained consistently in the neighborhood of 10,500°, while in the second, Williams (*M.N.*, **95**, 573, 1935) derives an absolute magnitude at maximum of -6.5 from a series of measures of the intensity of the interstellar K line. If we adopt Petrie's value of the temperature, then for a radius of 100×10^6 km we find $M_{\text{vis}} = -8.0$, which, judged by Williams' result, seems almost certainly too bright. There are at least three possible causes for this disagreement: (*a*) uncertainties in Williams' determination of the distance, owing largely to the high galactic latitude; (*b*) probably still greater uncertainties in the present estimate of size, inherent in the method; and (*c*) the necessity of assuming that the nova radiates as a black body. As regards (*c*), it is now known, through the work of Pannekoek (*ibid.*, p. 529), that the color temperatures of the stars in general may deviate widely from their effective temperatures. Accordingly, for this reason alone the foregoing computation of M may have little or no significance.

A NEW ORBIT FOR 29 CANIS MAJORIS

W. J. LUYTEN AND E. G. EBBIGHAUSEN

ABSTRACT

A new orbit for 29 Canis Majoris has been derived from twenty-four plates taken at the Yerkes Observatory. The elements as calculated from the absorption lines are found to be essentially the same, except for the eccentricity, as those determined by Harper in 1917; especially there appears to be no observable change in ω. In agreement with what has been found in other stars of similar spectral class, the emission lines of *He* II and *N* III show a persistent displacement toward the red of about 100 km/sec. with respect to the absorption lines.

I. INTRODUCTION

During a search for likely cases of spectroscopic binaries where a rotation of the line of apsides might be detectable, the star 29 Canis Majoris soon singled itself out by its promising characteristics: it is one of the very few class O binaries of high mass with a reasonably large eccentricity, while a good orbit by Harper[1] is available, determined as early as 1917. Its spectrum shows the further interesting feature of emission lines of *He* II and *N* III. As in the case of α Virginis, Dr. Struve kindly consented to have the star reobserved during the 1934–1935 season.

II. THE OBSERVATIONS

Owing to the southern declination of the star and the bad observing conditions usually prevailing during the short season, only twenty-four spectrograms could be secured. While this number is insufficient for the calculation of an accurate orbit, approximate values for the elements may be derived and a possible change in ω deduced.

All plates were measured twice by Ebbighausen; for the absorption lines the following were usually measured: $\lambda\lambda$ 4026.189, 4088.863, 4101.738, 4097.327, 4340.467, 4471.477, 4541.612, 4861.326, while the emission line of *He* II (4685.74) was measured on eighteen plates and those of *N* III (4634.145 and 4640.631) on six plates. The measured velocities are collected in Table I.

Harper derived a period of 4.3934 days, which was later corrected

[1] *Pub. Dom. Obs.*, **4**, 115, 1917.

by Pearce[2] to 4.39351. This value has been used to reduce all observations to one period. Subsequently, a comparison of our observations with those of Harper showed that Harper's period is more nearly correct; the influences of the difference between the two upon the phases of the present series of observations (which cover only twenty-three periods) is negligible and no recalculation of phases has been made.

TABLE I

YERKES MEASURES

Date	Julian Date 2420000+	Velocity in Km/Sec.		Date	Julian Date 2420000+	Velocity in Km/Sec.	
		Absorption Lines	Emission Lines			Absorption Lines	Emission Lines
1931 Feb. 25	6397.673	−183*	−117	1935 Jan. 14	7816.833	−201	−175
				22	7824.660	− 51	+164
1934 Dec. 5	7776.903	−186	− 61	Feb. 13	7846.738	− 42	+ 98
11	7782.864	− 46	......	21	7854.639	+186	+367
24	7795.788	−126	......	Mar. 2	7863.622	+179	+339
31	7802.852	−191*	+ 33	11	7872.633	+ 97	+254
				15	7876.573	+207	......
1935 Jan. 3	7805.751	+205	+297	16	7877.638	−110	+ 15
3	7805.833	+197	......	24	7885.569	+192	......
4	7806.836	+ 56	+226	27	7888.554	+ 43	+ 87
5	7807.797	−200	− 86	29	7890.578	− 17	+170
11	7813.820	+ 44	+134	Apr. 7	7899.576	−114	+ 5
14	7816.760	−212	−147	13	7905.577	−124	......

III. THE ORBIT (ABSORPTION LINES)

Owing to very large deviations from the velocity-curve, the two absorption-line velocities marked with an asterisk were omitted; the remaining twenty-two were grouped into eleven normal places and the orbital elements were calculated by the Wilsing-Russell method. These elements are given in the third column of Table II, while those published by Harper are added in the second column. The agreement may be considered satisfactory, except for the eccentricity. Since there is a possibility that the difference in e may have been caused by the different methods of calculating the elements, new solutions were made from Harper's observations. First, the same

[2] *Pub. Dom. Ap. Obs.*, **6**, 50, 1932.

normal places were used with the Wilsing-Russell method to calculate the elements given in the fourth column; it will be seen that the eccentricity has still further increased, while there is no noticeable change in any of the other elements. An inspection of Harper's original data revealed two discordant observations; if these are rejected, still another set of elements may be calculated, which are given in the fifth column of Table II. The eccentricity, though somewhat smaller than before, differs radically from that found by us. Furthermore, K has been reduced by 8 km/sec. From an observational point of view there can be little or no justification in this arbitrary rejection of data; on the other hand, we find it hard to believe

TABLE II

ORBITAL ELEMENTS

	Harper Original	Yerkes	Harper I	Harper II
K.......	218.4 ±3.1	220.9 ± 4.3	219.2 ±5.0	211.0 ±4.6
e........	0.156± .017	0.077± .019	0.174± .014	0.138± .022
ω	37.6 ±5.0	29.1 ±13.7	36.5 ±8.0	25.4 ±9.0
γ..	−12.1 ±2.3	−9.0 ± 3.0	−8.0 ±3.5	−7.0 ±3.5

$P = 4.3934$ days

in the reality of such a change in the eccentricity, simply because there is no adequate dynamical reason for it. In the absence of such, our procedure may be considered solely an attempt to see how much of the difference could possibly be ascribed to the observations. There still remains the possibility of differences due to the emulsion of the plates and the resolving power of the instrument in connection with blended lines, etc. Until all such sources of differences have been investigated and eliminated, we are not justified in interpreting the change in the calculated value of e as a real secular variation of the eccentricity.

It may be mentioned that the K line appears to give a constant velocity of $+29$ km/sec.

IV. THE EMISSION-LINE ORBIT

It became apparent from the earliest measurements that there exists a large systematic difference between the absorption-line and

the emission-line velocities. Even though velocities were obtained from only eighteen plates, these were used to derive a separate orbit whose elements were found to be

$$K = 248 \pm 8 \text{ km/sec.}, \quad \omega = 69^\circ \pm 30^\circ,$$
$$e = 0.093 \pm .030, \quad \gamma = +100 \pm 6 \text{ km/sec.}$$

Considering the scantiness of the material, the values for e and ω may well be identical with those derived from the absorption lines as given in Table II; the velocity of the system, however, is now 109 ± 7 km/sec. more positive. Furthermore, the larger value found for K (29 km/sec. larger than for the absorption lines) appears to be real.

Harper, in discussing the emission lines on his plates, states that "they partake of the periodic displacements due to the star's orbital motion just as do the absorption lines."[3] Dr. Struve calls attention to the fact that Harper determined the wave-length of his single emission line in such a way as to obtain the best possible fit between emission and absorption lines. The wave-length thus used is 4687.54 A on Rowland's system or 4687.36 on the International system while that now generally adopted[4] is 4685.74 A. The resulting correction to Harper's velocities amounts to +102 km/sec. Recalculation of the elements then yields

$$K = 235 \pm 12 \text{ km/sec.},$$
$$e = 0.048 \pm .111,$$
$$\gamma = +95 \pm 10 \text{ km/sec.}$$

ω is naturally indeterminate. The other values are in close agreement with those determined at Yerkes.

In both emission-line orbits there appears to be a slight "lag" as against the absorption lines, amounting to 0.16 days for the Yerkes measures and 0.06 days for Harper—the latter value, however, being less than its uncertainty.

Dr. Struve further calls attention to the fact that measures of the *He* II 4686 line, when it occurs in emission in O-type stars, nearly

[3] *Op. cit.*, p. 122.

[4] *Trans. I.A.U.* **4**, 1932.

always indicate[5] a red displacement of one or two angstrom units. Accordingly, the red shift of this line does not seem to be a peculiarity of 29 Canis Majoris, nor does it seem to be related to orbital motion. No lines are known in the vicinity of λ 4686 that could give rise to a blend; the orbital elements as given here were determined entirely from the *He* II line. However, on the best plates the emission lines of *N* III definitely partake of the large displacement toward the red.

V. THE APSIDAL MOTION

The three different solutions indicate that for Harper's observations at mean epoch 1916.4, ω must have been close to 30°, with a mean error of about 8°, while in the more recent Yerkes measures, of mean epoch 1935.1, ω is likewise found equal to $30^\circ \pm 14^\circ$, the larger mean error being due to the smaller eccentricity. Before concluding that there has been no observable change in ω, the possibility that the line of apsides has made one or more whole revolutions should at least be considered, even though such a rapid motion appears a priori unlikely.

Fortunately, the observations of Pearce, though few in number, are helpful in this connection. If the line of apsides had made one complete revolution forward or backward in the 18.7 years between the Ottawa and the Yerkes measures, then ω should have been in the third quadrant at Pearce's epoch; but the observations give no such indication. On the contrary, they suggest a value of ω which is essentially the same as that for the other two series. This state of affairs is compatible only with a moving apse if the period of apsidal revolution is at most 9.4 years. Since Harper's observations cover an interval of 104 orbital revolutions, ω should have changed by at least 24° in this interval, in which case one would not expect Harper's observations to have shown a larger eccentricity than the present Yerkes series.

The only reasonable conclusion, therefore, seems to be that no observable change in the direction of the line of apsides has taken place.

[5] Plaskett, *Pub. Dom. Ap. Obs.*, **2**, 16, 1930; Beals, *ibid.*, **4**, 9 and 17, 1934.

VI. GENERAL REMARKS

In measuring the lines of both components on the Victoria plates, Pearce has been able to derive minimum values for the semi-axis major of the relative orbit and the individual masses:

$$a \sin i = 30.1 \times 10^6 \text{ km} , \qquad m_2 \sin^3 i = 24.3 \odot ,$$
$$m_1 \sin^3 i = 32.2 \odot , \qquad m_2/m_1 = 0.75 .$$

In so far as we are aware, no variation in brightness has ever been observed in this star, but owing to its southern declination the variation, if any, might still be $0^{m}.5$ or more. Nothing very definite can therefore be concluded concerning the inclination, beyond that it is probably less than 80°. Where the density of the O stars is normally of the order of 0.01 of that of the sun, the radii of the components will probably be of the order of 10^7 km, or about one-third of the orbital radius. Under these conditions one would expect strong tidal forces, and, on either Russell's or Walter's theory, a rapid rotation of the line of apsides. If we are justified in our conclusion that there has been no observable change in 19 years, then it is also indicated that the theory of apsidal motion is considerably more complicated than has been believed heretofore.

MINNEAPOLIS
July 18, 1935

A TEST OF THERMODYNAMIC EQUILIBRIUM IN THE ATMOSPHERES OF EARLY-TYPE STARS

BY OTTO STRUVE

ABSTRACT

The relative intensities of high-level and of low-level *O* II lines, in B and O stars, change with temperature. An application of the Boltzmann formula gives reasonable differences of temperature. The intensities of *He* I lines show variations which are contrary to Boltzmann's formula, and Rudnick has shown that this effect cannot be explained by turbulence. It is suggested that the *He* anomaly results from an accumulation of atoms in the triplet system, the lowest term of which is metastable. Such an accumulation is possible if there are departures from thermodynamic equilibrium, e.g., if the photospheric radiation is appreciably diluted in the absorbing atmosphere. Departures of this nature are not improbable in the giants of early type.

EXCITATION OF *O* II

Theoretical investigations of stellar atmospheres are usually based upon the assumption that the conditions of excitation and ionization are as they would be in thermodynamic equilibrium. For interstellar matter and for nebulae, this assumption cannot be made, and Eddington[1] has shown that the stage of ionization of interstellar gas of density δ is the same as that of a gas having a density of δ/β, where[2]

$$\beta = \frac{R^2}{4r^2},$$

and where R is the radius of the star and $r >> R$ is the distance between the gas and the center of the star. It is obvious that consistent results for the state of ionization in a gas do not form a sufficient condition for the existence of thermodynamic equilibrium.

In the spectra of early-type stars ionization dominates spectral type. This may be inferred from the small amount of scatter when the ionization potential is plotted against that spectral class in which the lines of a given element have their maximum intensity.[3] The differences in excitation potential for the lines of any given atom are usually small, and differences in excitation have little effect upon the observed maximum.

[1] *The Internal Constitution of the Stars* (German ed.), p. 478, 1928.

[2] Rosseland, *Astrophysik auf Atom-theoretischer Grundlage*, p. 228, 1931.

[3] See, e.g., C. H. Payne, *M.N.*, **92**, 377, 1932.

Accordingly, we have as yet no real test of our basic assumption of thermodynamic equilibrium. That such a test would be useful is suggested by the increasing amount of data concerning peculiar stellar atmospheres which are enormously large in extent. I refer to such objects as ζ and ϵ Aurigae, P Cygni, 17 Leporis, etc. Even in well-known supergiants such as β Orionis and α Cygni, the P Cygni character of $H\alpha$ suggests extended atmospheres. It is quite conceivable that such atmospheres are no longer even approximately in thermodynamic equilibrium and that they should really be classed as intermediate between nebulae and normal reversing layers. Indeed, the spectrum of P Cygni seems to present evidence against thermodynamic equilibrium.[4]

TABLE I

Wave-Length	Lab. Int.	Theor. Int.	Designation	Excit. Pot. (Volts)
4303.82	5n	....	$x^4P_{2\frac{1}{2}}-f^4D^0_{3\frac{1}{2}}$	28.70
4275.52	4n	....	$z^4D_{3\frac{1}{2}}-e^4F^0_{4\frac{1}{2}}$	28.73
4317.16	8	40	$y^4P_{\frac{1}{2}}-e^4P^0_{1\frac{1}{2}}$	22.87
4319.65	8	43	$y^4P_{1\frac{1}{2}}-e^4P^0_{2\frac{1}{2}}$	22.88
4325.77	3	8	$y^4P_{\frac{1}{2}}-e^4P^0_{\frac{1}{2}}$	22.87
4345.57	7	40	$y^4P_{1\frac{1}{2}}-e^4P^0_{\frac{1}{2}}$	22.88
4349.44	8	100	$y^4P_{2\frac{1}{2}}-e^4P^0_{2\frac{1}{2}}$	22.90
4366.91	7	43	$y^4P_{2\frac{1}{2}}-e^4P^0_{1\frac{1}{2}}$	22.90

In this paper we shall make a preliminary test of the Boltzmann law in *O* II and *He* I. Russell and Adams[5] have made a similar test for *Fe* I and for other elements in late-type stars, and have found appreciable departures from thermodynamic equilibrium. Unfortunately, the range in excitation potential for *O* II is not sufficient to apply their method, and we shall therefore use only *O* II lines having low and high potentials. I have chosen the lines shown in Table I. The first two lines have high excitation potentials, while the rest have low excitation potentials.

The plates show clearly that the high-excitation lines are relatively stronger in the hotter stars of each of the two sequences. In Table II estimates are given of the line intensities derived on a uniform system.

[4] *Ap. J.*, **81**, 66, 1935.

[5] *Ibid.*, **68**, 19, 1928.

In order to evaluate these estimates, I have used the theoretical intensities of multiplet $y^4P - e^4P^\circ$. For each star the estimated intensities are plotted against laboratory intensities and they form a crude reduction-curve for the transformation of the estimates into relative numbers of atoms. A difficulty is at once obvious: the lines

TABLE II

Star	High Excit.		Low Excit.						H.D. Sp.
	4304	4276	4317	4320	4326	4346	4349	4367	
	Dwarfs								
10 Lac	3	2	2	2	2	3	6	3	Oe5
υ Ori	3	2	4	4	2	3	7	2	B3
τ Sco	5	5	6	6	3	5	10	5	B0
β Cep	3	2	8	8	3	7	8	7	B1
δ Cet	2	1	8	8	2	4	7	7	B2
γ Peg	2	2	7	6	2	4	7	6	B2
ζ Cas	2	0	7	7	1	2	3	6	B3
θ Oph	2	2	5	4	1	3	4	4	B3
ϵ Cas	.....	.....	2	2	.....	.....	.....	0	B3
	Giants								
S Mon	1	0	1	1	0	0	1	0	Oe
λ Ori	2	1	1	1	0	2	4	1	Oe5
κ Ori	4	3	8	8	3	5	12	7	B0
ζ Per	3	3	10	10	2	5	12	7	B1
o^2 CMa	0	0	3	3	0	2	3	2	B5p

λ 4346 and λ 4349 are somewhat affected by the wing of $H\gamma$, especially in the cooler dwarfs.

For several stars I have obtained accurate microphotometer records and these substantiate the relative change in the intensities of high-level and of low-level lines.

Boltzmann's formula gives for the number of atoms in energy state s:

$$\frac{n_s}{n} = Ce^{-\frac{11{,}600 E_s}{T}},$$

where E, the energy, is expressed in volts and T is in degrees. Accordingly, for two stars we have

$$\log n_s = \log n + \log C - 5040 E_s \left(\frac{1}{T}\right),$$

$$\log n'_s = \log n' + \log C - 5040 E_s \left(\frac{1}{T'}\right).$$

By subtraction:

$$\log \frac{n_s}{n'_s} = \log \frac{n}{n'} - 5040 E_s \left(\frac{1}{T} - \frac{1}{T'}\right).$$

If we have another energy state, t, we get

$$\log \frac{n_s}{n'_s} + \log \frac{n'_t}{n_t} = 5040 (E_s - E_t) \left(\frac{1}{T'} - \frac{1}{T}\right).$$

In our case, $E_s - E_t = 5.8$ volts. The values n_s/n'_s and n_t/n'_t are obtained from the calibrations of the estimated intensities for λ 4304 and λ 4317. If the two stars under consideration are 10 Lacertae and θ Ophiuchi, we get roughly

$$\frac{n_s}{n'_s} = 3 ; \qquad \frac{n'_t}{n_t} = 6 .$$

Assuming a temperature for θ Ophiuchi, we derive the corresponding temperature of 10 Lacertae:

T' (θ Oph)	T (10 Lac)	T' (θ Oph)	T (10 Lac)
10,000	17,000	15,000	40,000
12,000	25,000	23,500	∞

The results are in fair agreement with the ionization-temperature scale, and they support, as far as they go, the assumptions made by Fowler and Milne. However, no stress can be laid upon the numerical results, as a small error in the intensities would produce a marked change in the temperatures.

Quite similar results are obtained for the giants: the temperatures appear to be roughly the same as those of the dwarfs. The method is not accurate enough to register the temperature difference between giants and dwarfs, but it disproves, I believe, the low temperatures

obtained for B and O giants by spectro-photometric methods. Were there not already enough evidence, this would indicate that spectrophotometric gradients measure not so much the temperatures of these distant stars as the selective action of some unknown agent—possibly that of interstellar matter.

A useful test of Boltzmann's formula could probably be made for the following lines of *Si* IV:

Wave-Length	Lab. Int.	Excit. Pot.
4089	10	24 volts
4116	8	24 volts
4212	3	36 volts

Here $E_s - E_t = 12.0$ volts, and the effect should be more pronounced than in *O* II. However, λ 4212 is always much weaker than λ 4089, consequently it is much more difficult to eliminate the gradient effect. Qualitatively, the Boltzmann effect is present in the giants. Thus, in S Monocerotis λ 4212 is almost as strong as λ 4116, while in ζ Persei λ 4116 is much stronger. In the dwarfs, λ 4212 can be seen only between 10 Lacertae and τ Scorpii, and within this narrow range of spectral class there is no certain change in the relative intensities of the *Si* IV lines.

THE *He* I ANOMALY

Several years ago I called attention to the fact that the relative intensities of *He* I singlets and triplets are not the same in all stars.[6] Later I was inclined to attribute at least a part of this effect to turbulence.[7] However, a recent investigation by Rudnick[8] proves that the *He* I anomaly is not fully explained by turbulence. Consider, for example, the giant λ Orionis and the dwarf 10 Lacertae. Both are of Harvard class Oe5. In the giant the singlets are relatively much weaker than in the dwarf. The gradient can be estimated in both stars from the relative intensities of the *O* II and *Si* IV lines. It accounts for only a small part of the *He* I anomaly.

[6] *Nature*, **122**, 994, 1928; *Ap. J.*, **74**, 248, 1931.

[7] *Ap. J.*, **78**, 86, 1933; *ibid.*, **79**, 414, 1934; Russell, Mrs. Gaposchkin, and Menzel *ibid.*, **81**, 114, 1935.

[8] Unpublished.

Following is a summary of my results for the *He* I intensities:

1. The singlets and triplets reach maximum intensity in the same spectral subdivision, B2.
2. Near B2 the singlets are in all stars (giants and dwarfs) almost as intense as are the triplets.
3. In the earlier classes the singlets fade out more rapidly than the triplets.
4. This effect is especially pronounced in the giants.
5. There is probably a similar fading-out of the singlets in the later subdivisions, but the effect is less pronounced than in the B0 and O stars.
6. There is no appreciable difference, in the later types, between giants and dwarfs.

The excitation potentials of the singlets are about 0.3 volt higher than those of the triplets. Accordingly, the fading of the singlets in the O stars cannot be caused by temperature.

In order to explain the *He* anomaly it is useful to consider the following points: (*a*) the anomaly has been observed only in *He* I, in which there are almost no intersystem lines and in which the lowest 2^3S-level is metastable; (*b*) the anomaly is greatest in the hottest stars, where *He* I is mostly ionized, and recombination must be powerful. I should like to suggest that there are large departures from thermodynamic equilibrium in the atmospheres of early-type giants, and that the radiation from the photosphere is appreciably diluted by the factor β. We can then apply a method similar to one used by Rosseland.[9] Let there be three states of the atom and let a_{ik} be the corresponding transition probabilities, and x_i the relative numbers of atoms in the three states. If state 2 is metastable

$$a_{12}=a_{21}=0\ .$$

As a first approximation we neglect collisions in the giants. We may write:

$$\frac{x_2}{x_1}=\frac{a_{13}a_{32}}{a_{31}a_{23}}=\frac{a_{13}\rho_{13}a_{32}}{a_{31}a_{23}\rho_{23}}\ ,$$

[9] *Loc. cit.*

since

$$a_{ik} = \alpha_{ik}(1+\rho) \text{ for emission },$$

$$a_{ki} = \alpha_{ik}\, \rho \frac{\omega_i}{\omega_k} \text{ for absorption },$$

$$\rho_{13} = \beta \left(e^{\frac{h\nu_{13}}{kT}} - 1 \right)^{-1},$$

$$\rho_{23} = \beta \left(e^{\frac{h\nu_{23}}{kT}} - 1 \right)^{-1}.$$

These expressions hold for line absorption, but Woolley and others have extended the reasoning to include the state of ionization. Accordingly, we shall assume that state 1 is the 1^1S level of *He* I, state 2 the 2^3S level, and state 3 is that of ionization. In our case it will suffice to assume that the continuous absorption at the series limit is effectively concentrated within a fairly narrow range of frequencies, so that ν_{13} and ν_{23} have a definite meaning. Our expressions, then, simply indicate that the numbers of ionizations are proportional to the energy density, while the recombinations are independent of the radiation:

$$\frac{x_2}{x_1} = \text{Const.} \frac{e^{\frac{-h\nu_{12}}{kT}} - e^{\frac{-h\nu_{13}}{kT}}}{1 - e^{\frac{-h\nu_{13}}{kT}}}.$$

Accordingly, as a first approximation, x_2/x_1 is independent of β and resembles the value given by Boltzmann's formula if $\nu_{13} >> \nu_{12}$.

Now consider for state 2 some other singlet level of *He* I, for example, 2^1P. Evidently now: $a_{12} \neq 0$ and $a_{21} \neq 0$. According to Rosseland, in this case

$$\frac{x_2}{x_1} \approx \beta e^{-\frac{h\nu_{12}}{kT}}.$$

In the hotter giants, where collisions are negligible, where β is appreciable, and where a_{13} is large, there should be an accumulation of atoms in the triplet system: the triplet lines should appear relatively stronger than the singlet lines. In the dwarfs, β is small and collisions are more frequent. Consequently, thermodynamic equilibrium

should be more nearly preserved, and Boltzmann's formula should be more closely obeyed than in the giants.

In the cooler stars a_{13} is relatively small, and recombination is not as important as in the hotter stars. A quantitative treatment could be obtained by introducing, in addition to a_{13}, a_{31}, a_{23}, a_{32}, six probabilities of collision, c_{12}, c_{21}, c_{13}, c_{31}, c_{23}, c_{32}, for which the metastability of state 2 does not exist. These values are smaller in the giants than in the dwarfs. If a_{13}, a_{31}, and a_{32} are large, the collision probabilities may be neglected. But if ionization does not take place, the c's are important and produce normal relative intensities in the triplets and singlets of the cooler giants.

Near class B2 the approximate equality of singlets and triplets is probably brought about by the gradient effect. The curves of growth of lines broadened by Stark effect have not been investigated, but it is probable that Stark effect will influence the gradient as does turbulence.

In O II, intersystem lines are frequent and strong. Consequently we should not expect a marked anomaly in the giants. For two states, 1 and 2, we have

$$\frac{x_2}{x_1}=\beta\left(e^{\frac{h\nu}{kT}}-1+\beta\right)^{-1}=\beta e^{-\frac{h\nu}{kT}}\left[1+(1-\beta)e^{-\frac{h\nu}{kT}}+\ \cdots\ \right]\approx\beta e^{-\frac{h\nu}{kT}}\,.$$

Accordingly, the distribution of atoms will resemble that given by Boltzmann's formula, except for the factor β. The proportionality with β is retained if there are three states and if all transitions are possible.[10] If the second level is metastable, we get Rosseland's[10] result. If the third level cannot be reached directly from the first, x_3/x_1 is proportional to β^2 (provided there is no ionization and recombination).

In the foregoing discussion we have neglected interlocking of spectral lines, produced by the depletion of the continuous spectrum in the line frequencies. Eddington[11] has pointed out that the complete problem, even for three states, is intractable. We are, therefore, able to obtain only the following qualitative results: (a) the apparent verification of the Boltzmann formula for O II in giant spectra is no

[10] *Loc. cit.*

[11] *M.N.*, **89**, 635, 1929.

proof of thermodynamic equilibrium; (*b*) the *He* anomaly is almost entirely produced by large departures from thermodynamic equilibrium.

The laboratory results relating to excitation functions for *He* I deal with electronic collisions and are, therefore, not similar to results derived from stellar spectra, where excitation by radiation is important.

The emission lines of *He* I in nebulae, in P Cygni, in β Lyrae, etc., agree well with the theory: the singlets are exceptionally weak. The absorption lines in β Lyrae also show the anomaly. In the absorption lines of P Cygni the anomaly is less pronounced. I suspect that this is due to turbulence, which brings the strong lines of *He* I near the horizontal portion of the curve of growth.

There are two serious objections to our theory which cannot be satisfactorily met at the present time: (*a*) our test for singlets and triplets depends almost exclusively upon the diffuse series of each system; (*b*) within each series we have compared only two or three lines.

Whether other singlet and triplet lines will give similar results remains to be seen. Most of these are weak, or they lie outside the best region of our plates. The line (2^1S-4^1P), λ 3965, is very weak in S Monocerotis. It is strong, however, in β Lyrae and P Cygni. The fact that we consider only one or two members in each series leaves open the question whether the intensity gradient is constant in each series. We know that stellar Balmer lines, in absorption, do not have a constant gradient.

YERKES OBSERVATORY
April 30, 1935

THE EFFECT OF REFLECTION UPON THE PROFILES OF ABSORPTION LINES IN SPECTROSCOPIC BINARIES

By E. L. McCARTHY

ABSTRACT

The unsymmetrical profiles of absorption lines caused by axial rotation and reflection in close spectroscopic binaries were computed for α_2 Vir, V_2 Pup, and u_2 Her. The resulting errors in the radial velocities were evaluated for various phase angles. At quadrature they are approximately 2 km/sec. for α_2 Vir; 18 km/sec. for V_2 Pup; and 14 km/sec. for u_2 Her.

1. In the profiles of spectral lines broadened by rapid axial rotation each point corresponds effectively to a definite portion of the star's disk. Accordingly, if the distribution of light over the star is not uniform, the profile will not be symmetrical, and a study of the degree of asymmetry should lead to a determination of the light-distribution on the surface of the star. Struve[1] suggested that such unsymmetrical lines should be produced by the effect of reflection in close spectroscopic binaries.

It is the purpose of this paper to evaluate the amount of asymmetry, from theoretical considerations, for three typical binaries and to investigate the resulting change of the radial velocity as determined from the unsymmetrical lines. At full phase the increase in total brightness, due to reflection, is given by Eddington[2] as

$$I_0=\frac{17}{24}\frac{L_1}{L_2}\left(\sin^2\varphi+\frac{2+\cos^3\varphi-3\cos\varphi}{\sin\varphi}\right),$$

where I_0 gives the increase in brightness as a fraction of the normal brightness, L_1/L_2 is the luminosity ratio of the illuminating star to the reflecting star and $\sin\varphi=R^2/a$ is the ratio of the radius of the reflecting star to the distance between the centers of the two components.

Evidently the reflection effect will be largest for spectroscopic bi-

[1] *Observatory*, **57**, 274, 1934.

[2] *The Internal Constitution of the Stars*, p. 212, 1926.

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[1] *Observatory*, **57**, 274, 1934.

[2] *The Internal Constitution of the Stars*, p. 212, 1926.

naries of short period and of large amplitude in velocity. For the three stars shown in the accompanying table the orbital elements give the quantities L_1/L_2, sin φ, and the equatorial velocity of rotation (the subscript 2 in the names of the stars designates the fainter component of the system). The inclination was assumed to be 90°,

Star	sin φ	L_1/L_2	Equatorial Velocity
α_2 Vir	0.20	6.25	70 km/sec.
V_2 Pup	.42	1.5	262
u_2 Her	0.40	2.5	145

which is probably a good approximation in the case of V Puppis and u Herculis, and is probably also reasonably close to the truth for α Virginis, which is not definitely known to be an eclipsing variable. The rotational profiles were computed for a standard line having the same shape as that of *Mg* II λ 4481 in α Cygni as measured by Elvey. This non-rotational profile has a maximum absorption of 71 per cent and a total absorption of 0.80 A. The computations were made for the following phases:

$$\psi = 0\,, \quad \frac{\pi}{4}\,, \quad \frac{\pi}{3}\,, \quad \frac{\pi}{2}\,, \quad \frac{2\pi}{3}\,, \quad \frac{3\pi}{4}\,, \quad \text{and} \quad \pi\,.$$

At $\psi = 0$ there is no reflected light and the profile is normal. At $\psi = \pi$ the entire disk is illuminated and the profile is again symmetrical except for the effect of the eclipse, which has not been considered here since it has been discussed elsewhere.[3] For all intermediate phases the profile is unsymmetrical. If the non-rotational profile is narrower than that observed in α Cygni, the effects would be slightly larger than those computed by me. Darkening at the limb has been neglected, but it would probably not materially change the results.

2. In order to obtain an upper limit for the effect of reflection, the assumption was first made that the entire illuminated part of the apparent disk of the star has a constant brightness equal to $(1 + I_0)$.

[3] Struve and Elvey, *M.N.*, **91**, 663, 1931.

Through the method of graphical integration used by Shajn and Struve[4] and by Elvey,[5] the rotational profiles for the three stars were computed as shown in Figures 1, 2, and 3. The stars were assumed to be spherical in shape. The profile of asymmetry is negligible in the case of α Virginis but it is quite pronounced for V Puppis and u Herculis.

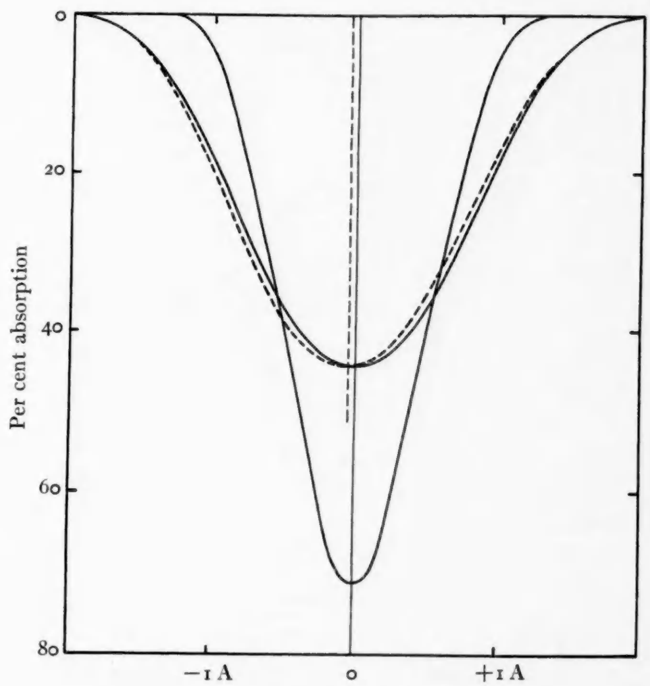

FIG. 1.—Computed line profiles of α_2 Virginis

3. The error in radial velocity may be determined by assuming that the setting is made in such a way that the areas of the violet and the red parts of the profile are equal. The corresponding displacements obtained by means of a planimeter are given in column *a* of Table I. The maximum error for α Virginis is 2.5 km/sec. while for V Puppis it is 18 km/sec. and for u Herculis it is 14 km/sec. The dotted lines in the figures show these displacements.

[4] *Ibid.*, **89**, 567, 1929. [5] *Ap. J.*, **71**, 221, 1930.

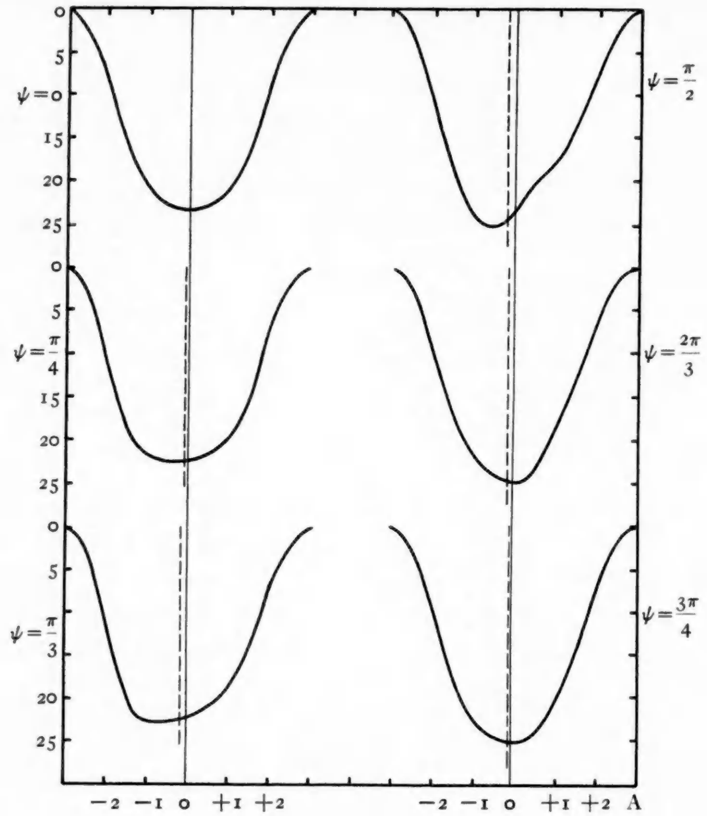

FIG. 2.—Computed line profiles of u_2 Herculis

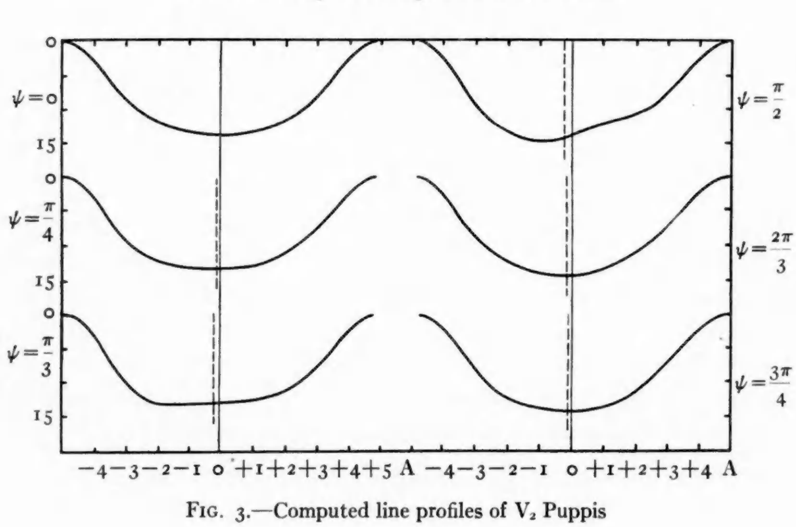

FIG. 3.—Computed line profiles of V_2 Puppis

4. A simple expression for the error in wave-length may be obtained under the assumption that the setting is made in the center of gravity of the profile. For an intermediate phase angle ψ, the light from each component of the binary may be considered as coming from two separate sources: A full disk whose brightness is normal and an area having a crescent or gibbous shape which is equal to $0.5\,(1-\cos\psi)$ times the area of the disk, and whose brightness is I_0 times the normal surface brightness of the star. The combination of

TABLE I

ψ	α_2 Virginis			V_2 Puppis			u_2 Herculis		
	a	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
	km/sec.	km/sec.	km/sec.	km/sec.	km/sec.	km/sec.	km/sec.	km/sec.	km/sec.
0	0	0	0	0	0	0	0	0	0
$\pi/4$	....	1.2	0.4	7.8	6.7	2.3	6.1	5.5	1.9
$\pi/3$	....	1.8	0.8	14	9.8	4.4	11	7.9	3.6
$\pi/2$	2.5	2.3	1.5	18	12	8.2	14	9.7	6.6
$2\pi/3$	....	1.6	1.4	12	8.8	7.4	8.4	6.8	5.8
$3\pi/4$	....	1.1	1.0	7.0	5.7	5.2	4.7	4.4	4.0
π	0	0	0	0	0	0	0	0	0

Column *a* gives the error in radial velocity obtained from figures 1, 2, and 3; column *b* was computed from formula (1) and column *c* was computed from formula (2).

these two separate areas gives rise to a normal symmetrical profile of unit area at the true position of the line and to a smaller profile of area $I_0/2\,(1-\cos\psi)$ at a distance s from the true position. The center of gravity of the combined profile is the center of gravity of these components and evidently lies at

$$\Delta\lambda=\frac{I_0(1-\cos\psi)}{2+I_0(1-\cos\psi)}\,s\,.$$

It is necessary to evaluate s in terms of known parameters. Evidently the center of gravity of the profile which is due entirely to reflected light is displaced from the center of the normal line by an amount s corresponding to the radial velocity of the geometrical center of gravity of the reflecting area. If the equatorial velocity V is represented by the radius of the disk, then the reflecting area is

always bounded by a semicircle and an ellipse of a minor semiaxis $V \cos \psi$. The center of gravity of the figure lies at

$$s = \frac{4V}{3\pi}(1+\cos\psi)\ .$$

Using this expression, we find

$$\Delta\lambda = \frac{4VI_0}{3\pi}\left[\frac{1-\cos^2\psi}{2+I_0(1-\cos\psi)}\right]. \qquad (1)$$

The results of the computation of $\Delta\lambda$ are given in columns b of Table I.

5. In both the graphical and the analytical treatments it was assumed that the increase in surface brightness was given by I_0, regardless of the angle between the line of sight and the direction of the light incident upon the star. This assumption makes the values of $\Delta\lambda$ calculated by both methods undoubtedly too large, since there must be a considerable decrease in the intensity of the reflected light at increasing angles from the normal. In order to obtain a better approximation, the intensity of the reflected light may be assumed[6] to be proportional to

$$\frac{1}{\pi}(\sin\psi - \psi\cos\psi)\ .$$

This includes the loss in total brightness due to the increasing angle of reflection and to the reduced reflecting surface, which is $0.5\,(1-\cos\psi)$ times the area of the disk. The surface brightness of the reflecting area is therefore equal to

$$\frac{2I_0}{\pi}\left(\frac{\sin\psi - \psi\cos\psi}{1-\cos\psi}\right).$$

Using this expression, we obtain:

$$\Delta\lambda = \frac{4VI_0}{3\pi}\left[\frac{(1+\cos\psi)(\sin\psi - \psi\cos\psi)}{\pi + I_0(\sin\psi - \psi\cos\psi)}\right]. \qquad (2)$$

[6] Eddington, *loc. cit.*

The results obtained from formula (2) are given in columns c of Table I.

6. Table I shows that the error in radial velocity is large only when the rotational velocity is so great that the lines are diffuse and difficult to measure. In most cases they are just within the probable errors of the measurements. Thus Miss Maury[7] gives a probable error for her objective-prism measurements of V Puppis as ± 16 km/sec. If the maximum error in the radial velocity of V_2 Puppis is 8.2 km/sec., the corresponding error for V_1 Puppis would be approximately 5.3 km/sec. The measured separation between the two lines would, therefore, be 13.5 km/sec. less than the true separation. For u Herculis, Baker[8] has given $K_2 = 253 \pm 12$ km/sec., and $K_1 = 99 \pm 1.0$ km/sec. Table I indicates that K_2 is approximately 6.6 km/sec. too low and the corresponding value for K_1 is about 1.2 km/sec. The probable error from slit spectrograms is of the order of 4 or 5 km/sec. for stars with diffuse lines.

The error in radial velocity is not the same for complementary phases: Those in column c show an error of 4.4 km/sec. for V_2 Puppis at $\psi = \pi/3$. The corresponding error at $\psi = 2\pi/3$ is 7.4 km/sec. Theoretically, this should introduce a slight error into the eccentricity of the orbit but the effect is so small as to pass unnoticed. It is of interest to note that the effect upon the eccentricity is opposite in columns b and c of Table I.

The asymmetry at $\psi = \pi/2$ should be sufficient to be detected by means of microphotometer tracings. However, the changes in the intensities and profiles of the lines detected by Struve and Ebbighausen[9] in α Virginis and by Bailey and Miss Maury[10] in V Puppis are of an entirely different origin and probably would mask the relatively small effect caused by reflections.

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[7] *Harvard Annals*, **84**, 174, 1933.

[8] *Pub. of the Allegheny Obs.*, **1**, 82, 1910.

[9] *Ap. J.*, **80**, 365, 1934.

[10] *Harvard Ann.*, **84**, 169, 1920.

REVIEWS

Mathematical Problems of Radiative Equilibrium. By EBERHARD HOPF. ("Cambridge Tracts in Mathematics and Mathematical Physics," No. 31.) Cambridge: University Press; New York: Macmillan Co., 1934. Pp. viii+104. 6*s*. net.

The enormous variety of astrophysical phenomena brought to light by an ever increasing number of large telescopes and enthusiastic observers makes difficult the application of mathematical methods in all their rigor and beauty. These new facts stubbornly escape mathematical treatment, making it to a certain degree fruitless and tending to transform astrophysics into a natural science which describes the phenomena and makes generalizations of a purely qualitative character. A mathematically minded astrophysicist, educated in the traditions of classical astronomy, will never approve such tendencies; at best he will look upon them as temporary and inevitable evils. Such an astrophysicist would welcome a purely mathematical development of stellar hydrodynamics, a mathematical theory of polytropic structures, an abstract treatment of radiative equilibrium, etc. Investigations of this type do not always help a practical astrophysicist who is impatient to explain an enormous number of obscure phenomena. These investigations, although of no immediate use, slowly lay the foundation for the astrophysics of tomorrow which will be an exact science like classical astronomy.

Dr. Hopf's book, being of such a fundamental character, deals with the theory of radiative equilibrium, which has been developed by Schwarzschild, Milne, and others. It aims at a comprehensive representation of all that has been achieved in this field. It should be remembered that seven years ago Dr. Hopf proved the existence of a non-zero solution of the homogeneous integral equation of radiative equilibrium, which other investigators, who made use of the general methods of existence theorems, had failed to find. This and other papers on radiative equilibrium make Dr. Hopf one of the best-qualified persons to write a book on the subject.

In this work Dr. Hopf deals with fundamentals instead of exploring the unknown. This may disappoint some readers who expect to find the

solution of some unsolved or partially solved problems such as the spherical problem, the effect of a given distribution of spectral lines, the illumination of planetary atmospheres, etc. However, the author can hardly be held responsible for the scientific tastes of his readers.

The first chapter deals with the principles and the formulation of four main problems: the first two are connected with the Schwarzschild-Milne model (with and without the liberation of energy), and the other two are connected with Schuster's model (finite and infinite optical thickness). All of these problems are reduced to integral equations, the analysis of which is the main subject of the subsequent chapters. The solutions of Problems I and II are contained in the second chapter. It is shown that the integral equation of problem I, i.e., $B(\tau)=\Lambda(B)_\tau$, has a solution $f(\tau)=\tau+q(\tau)$ $(\frac{1}{2}<q<1)$, and that this non-negative solution is unique. It is further demonstrated that the non-homogeneous integral equation of problem II has a non-negative solution (represented by a Neumann series) only if the energy liberated in a normal column of cross-section one is finite. The Hopf-Bronstein theorem on the boundary temperature is then proved and the properties of $q(\tau)$ are studied. Next the Schwarzschild-Milne problem for non-zero incident radiation is investigated.

Schuster's problem has not been hitherto discussed in its full generality when the general law of scattering is assumed (only the simplest case of uniform scattering has been thus far investigated.) The discussion of problems III and IV in their more general aspects is contained in chapter iii. Here the corresponding existence theorems are proved and the limits for the *Ergiebigkeit* are given. Chapter iv deals with explicit solutions of certain more general integral equations, obtained by means of Fourier integrals. The resulting explicit, though very complicated, formula for the law of darkening is of particular interest to the astrophysicist. Other problems of radiative equilibrium are discussed in chapter v. The case of purely absorbing material with an absorption coefficient that varies with the wave-length is known to lead to a non-linear integral equation, which is briefly discussed in this chapter. The analysis of Milne's model of a planetary nebula in radiative equilibrium concludes the book.

Dr. Hopf's tract is of a purely mathematical character and those who are not familiar with integral equations will hardly be able to follow the author in all the intricacies of his numerous theorems and lemmas. The book does not open new vistas, nor does it make new pathways in science. Its value is of quite a different nature. By transforming the theory of

radiative equilibrium into a chapter on the theory of integral equations it is an important step toward the foundation of the mathematical astrophysics of the future.

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Etude de la lumière du fond du ciel nocturne. By C. FABRY, J. DUFAY, and J. COJAN. Edition de la *Revue d'optique théorique et instrumentale*. Paris, 1934. Pp. 55. Fr. 10.

In 1901 Newcomb published an important article in the *Astrophysical Journal* under the title "A Rude Attempt To Determine the Total Light of All the Stars." He found that on the average each square degree of the night sky emits an amount of light equivalent to that of a star of magnitude 4.84, and this quantity he considered "as among the most important fundamental constants in astrophysics." We now know, largely through the work of Fabry, Van Rhijn, Dufay, and others, that only a small portion of this constant is produced by the combined effect of unresolved stars. Recent estimates suggest that the ratio of starlight to that of the sky (per square degree) is not more than one-third. The remainder consists of two spectroscopically different factors: the emissions produced in the atmosphere of the earth and a certain amount of continuous radiation which is slightly polarized in a plane passing through the sun and which is therefore probably related to the zodiacal light and to the *Gegenschein*. Finally, there may be a small contribution from starlight scattered by interstellar matter.

The little book under review consists of three communications made by the authors at a meeting of the Institut d'Optique at the Sorbonne on June 13, 1933. The first, by Fabry, is a short historical account of the study of the sky brightness. This is followed by a longer article by Dufay in which the author summarizes his own investigations and compares them to those of other workers. The last chapter, by Cojan, is a critical discussion of spectrographic equipment suitable for work on the night sky.

There are many interesting results that should be stimulating to other workers. For example, what is the extinction coefficient for an extended luminous object? Dufay shows clearly that a star is more affected by atmospheric scattering than is the extra-terrestrial brightness of the sky. But what will be the effect upon a large luminous object, such as the Zodiacal Light or the Milky Way? On a very thick night the direct beam of a star is reduced by true absorption and by scattering. The scattered

light is partly added to the brightness of the sky, and may, under favorable circumstances, form an appreciable luminosity around the star. In photometric measures both the absorbed and the scattered light are lost. In the case of an extended luminous area the scattered light is partly added to that of the object itself, thereby slightly reducing the effect of extinction. We are reminded here of Barnard's observations of the *Gegenschein:* "I have been struck with the fact that a sky not over transparent is just as good (I won't say better) for observing the *Gegenschein* as the clearest, darkest night. It therefore does not require the serene purity of a mountain atmosphere to see it well." The contrast is further reduced by the atmospheric radiation.[1]

Dufay gives an interesting account of spectroscopic observations of the night sky. The emission lines have been thoroughly studied, but their identifications are incomplete. Relatively little is known concerning the distribution of light in the continuous spectrum of the night sky. I believe Dufay has investigated this problem elsewhere, but he does not refer to it in the present article. The fact that the total brightness of the sky increases toward the horizon may be due to the emission lines of the atmospheric radiation. The presence of the Fraunhofer absorption lines fixes the spectral class, but does not define the distribution of energy in the continuous spectrum.

Of special interest to astronomers is the problem of interstellar scattering. Knowing the number of scattering particles and making reasonable assumptions as to their size, we can compute the approximate amount of light scattered by the stars. I made such a computation a few years ago and found a value that is somewhat in excess of that actually observed.[2] Considering the uncertainties of the problem and the simplifications made in the computation, this result is not in serious discord with the observations. Dufay points out that the amount of light per square degree contributed by an infinite scattering medium is exactly equal to that of

[1] See also C. Hoffmeister, *Veröffentlichungen Sternwarte Berlin-Babelsberg*, **8**, Part II, 23, 1930.

[2] In these computations I adopted for the number of particles per unit volume the value given by Gleissberg. It is surprising that the latter nowhere explicitly states the corresponding size of the particles. Evidently the method used by Gleissberg, and before him by Schönberg, is equivalent to the assumption that the size is such that Rayleigh's equation in the form used for gases is satisfied when the index of refraction for a solid (1.5) is substituted. By following the same method in my paper, I have virtually excluded the foregoing uncertainty: for any other size of particles the observational results would give another number per cubic centimeter, but the amount of scattered light should remain the same.

all stars as seen from the earth. This condition, he believes, may be approximately fulfilled in the Milky Way, but is certainly not true for the galactic poles.

O. STRUVE

The Structure of Spectral Terms. By W. M. HICKS. London: Methuen & Co., 1935. Pp. xi+209. 10*s.*, 6*d.*

The author gives an independent analysis of complex spectra, carried through without reference to current theories. No reasons are given for dissatisfaction with these theories and the terminology based upon them. The understanding of the book is rendered difficult by the absence of any correlation of the author's terms with those commonly accepted.

The validity of the analysis is rendered doubtful by a disregard of the statistical principles upon which all empirical conclusions must be based. A flagrant case of this disregard occurs on page 32. The fact that certain numbers are very often divisible by 4 has just been emphasized and a seventeen-page table is given in support of this conclusion, among others. There are forty entries on the first page, comprising all the examples taken from the spectra of *He*, *A*, *Kr*, and some of those from *X*. Of these forty numbers, nine are divisible by four, six have the remainder 1, eleven, the remainder 2, fourteen, the remainder 3. The same table would afford other instances of a similar nature.

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